Sikkim Public Service Commission

Written (Main) Examination for the post of
Sub-Jailer

Time Allowed: 3 hours

PAPER – II
MATHEMATICS

Maximum Marks: 250

INSTRUCTIONS TO CANDIDATES

Read the instructions carefully before answering the questions: -

1. **This Test Booklet consists of 12 (twelve) pages and has 63 (sixty-three) printed questions.**
2. **IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS BOOKLET DOES NOT HAVE ANY UNPRINTED, TORN OR MISSING PAGES OR ITEMS. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.**
3. **Use only Black Ball Point Pen to fill the OMR Sheet.**
4. Please note that it is the candidate's responsibility to fill in the Roll Number carefully without any omission or discrepancy at the appropriate places in the **OMR ANSWER SHEET** as well as on **SEPARATE ANSWER BOOKLET** for Conventional Type Questions. Any omission/discrepancy will render the Answer Sheet liable for rejection.
5. Do not write anything else on the OMR Answer Sheet except the required information. Before you proceed to mark in the OMR Answer Sheet, please ensure that you have filled in the required particulars as per given instructions.
6. This Test Booklet is divided into 3 (three) parts – **Part-I, Part-II** and **Part-III**.
7. All three parts are **Compulsory**.
8. **Part-I consists of Multiple-Choice Questions**. The answers for these questions have to be marked in the **OMR Answer Sheet** provided to you.
9. **Parts II and III consist of Conventional Type Questions.** The answers for these questions have to be written in the **Separate Answer Booklet** provided to you.
10. After you have completed filling in all your responses on the OMR Answer Sheet and the Answer Booklet(s) and the examination has concluded, you should hand over the OMR Answer Sheet and the Answer Booklet(s) to the Invigilator only. You are permitted to take the Test Booklet with you.
11. **Marking Scheme**
    THERE WIL BE **NEGATIVE MARKING** FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBECTVE TYPE QUESTIONS
    (i) There are four alternatives for the answer to every question. **For each question for which a wrong answer has been given by the candidate, one-third of the marks assigned to the question will be deducted as penalty.**
    (ii) If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to the question.
    (iii) If a question is left blank. i.e., no answer is given by the candidate, there will be no penalty for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

PART - I
(Multiple Choice Questions)

Choose the correct answer for Questions 1 to 50 from the given options. Each question carries 3 marks.
[50 x 3 = 150]

1. Let $L : R^3 \rightarrow R^2$ be the linear transformation, where $L(1,0,0) = (2,-1)$, $L(0,1,0) = (3,1)$ and $L(0,0,1) = (-1,2)$. Then $L(-3,4,2)$ is:
   (a) (-8, 3)
   (b) (4, 11)
   (c) (8, 11)
   (d) (16, 3)

2. Let $L : R^2 \rightarrow R^2$ be the linear transformation defined by $L\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} a_1 + 2a_2 \\ 2a_1 + 4a_2 \end{bmatrix}$.
   Then which of the vector is in kernel of L?
   (a) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$
   (b) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$
   (c) $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$
   (d) $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$

3. Let $L : R^4 \rightarrow R^6$ be a linear transformation. If $dim\ (kernel\ of\ L)\ is$ 2, then, $dim\ (range\ of\ L)$ is:
   (a) 6
   (b) 4
   (c) 2
   (d) 10

4. The eigen vector associated with the eigen value $\lambda = 2$ of the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ is:
   (a) $\begin{bmatrix} 2r \\ r \end{bmatrix}$
   (b) $\begin{bmatrix} r \\ 2r \end{bmatrix}$
   (c) $\begin{bmatrix} r \\ -2r \end{bmatrix}$
   (d) $\begin{bmatrix} r \\ -r \end{bmatrix}$

5. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which of the matrix is a zero matrix?
   (a) $A^2 - 7A - 6I = 0$
   (b) $A^2 + 7A - 6I = 0$
   (c) $A^2 - 7A + 6I = 0$
   (d) $A^2 - 7A + 14I = 0$

6. The eigen values of a skew-symmetric matrix are:
   (a) always pure imaginary
   (b) always non-zero real
   (c) zero or real
   (d) zero or pure imaginary

7. $\lim_{x \to 0} \frac{\sinh x - \sin x}{x \sin^2 x}$ is:
   (a) 1/3
   (b) 1/6
   (c) 1/2
   (d) 0

8. If $f(1) = 4$ and $f'(1) = 2$, then the value of the derivative of the function $\log f(e^x)$ w.r.t. x at $x = 0$ is:
   (a) 0
   (b) 1/2
   (c) 1
   (d) 2

9. The coefficient of $(x-1)^2$ in the Taylor's series expansion of $f(x) = x^2 e^x$ for all $x \in R$ about $x = 1$ is:
   (a) $\frac{3e}{2}$
   (b) $\frac{5e}{2}$
   (c) $\frac{7e}{2}$
   (d) $7e$

10. Let $f(x) = (x-2)^{13}(x+5)^{18}$.
Then:
(a) f does not have a critical point at $x = 2$
(b) f has a minimum value at $x = 2$
(c) f has a maximum value at $x = 2$
(d) f has neither a minimum nor a maximum value at $x = 2$

11. If $f(x,y) = \sum_{k=1}^{5}(x^2 - y^2)^k$ for all $(x,y) \in R^2$, then for all $(x,y) \in R^2$:
(a) $x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y} = 0$
(b) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 0$
(c) $y\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial y} = 0$
(d) $y\frac{\partial f}{\partial x} + x\frac{\partial f}{\partial y} = 0$

12. The volume bounded by the positive sides of the coordinate planes and the surface $\sqrt{x/a} + \sqrt{y/b} + \sqrt{z/c} = 1$ is
(a) $\frac{abc}{15}$
(b) $\frac{abc}{90}$
(c) $\frac{abc}{360}$
(d) $\frac{abc}{720}$

13. If $u = \frac{y^2}{x}, v = \frac{z^2}{y}$ and $w = \frac{x^2}{z}$, then the value of the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ is:
(a) 7
(b) -7
(c) 9
(d) -9

14. The volume of the tetrahedron formed by the plane surface $y + z = 0$, $z + x = 0$, $x + y = 0$ and $x + y + z = 1$ is:
(a) 1/6
(b) 1/3
(c) 2/3
(d) 4/3

15. The straight lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if:
(a) $ac' - a'c = 0$
(b) $ac' + a'c = 0$
(c) $aa' + cc' + 1 = 0$
(d) $aa' + cc' = 1$

16. The equation of the plane passing through the point (1, 2, -1) and containing the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z+2}{-1}$ is:
(a) $x - y - z = 0$
(b) $x + y - z - 2 = 0$
(c) $x + y + z + 2 = 0$
(d) $x - y + z + 4 = 0$

17. The equation of the sphere passing through the circles $y^2 + z^2 = 9, x = 4$ and $y^2 + z^2 = 36, x = 1$ is:
(a) $x^2 + y^2 + z^2 + 4x - 41 = 0$
(b) $x^2 + y^2 + z^2 + 4x + 41 = 0$
(c) $x^2 + y^2 + z^2 - 4x - 9 = 0$
(d) $x^2 + y^2 + z^2 + 4x - 9 = 0$

18. The equation of the cone passing through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}, \frac{x}{3} = \frac{y}{2} = \frac{z}{-1}$, is:
(a) $3yz - 10zx + 6xy = 0$
(b) $3yz + 10zx - 6xy = 0$
(c) $3yz - 10zx - 6xy = 0$
(d) $3yz + 10zx + 6xy = 0$

19. The equation of the curve passing through the point $(\pi/2, 1)$ and having the slope $\frac{\sin x}{x^2} - \frac{2y}{x}$ at each point (x, y), with $x \neq 0$ is:
    (a) $x^2y - \cos x = \frac{\pi^2}{4}$
    (b) $x^2y - \sin x = \frac{\pi^2+1}{4}$
    (c) $x^2y + \cos x = \frac{\pi^2}{4}$
    (d) $x^2y + \sin x = \frac{\pi^2-1}{4}$

20. The differential equation of the family of tangent lines to the parabola $y = x^2$ is:
    (a) $\left(\frac{dy}{dx}\right)^2 - 4x\frac{dy}{dx} - y = 0$
    (b) $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + 4y = 0$
    (c) $\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx} - 4y = 0$
    (d) $\left(\frac{dy}{dx}\right)^2 - 4x\frac{dy}{dx} + 4y = 0$

21. The solution of the differential equation $\left(y\left(1+\frac{1}{x}\right)+\cos y\right)dx + (x + \log x - x\sin y)dy = 0$ is:
    (a) $xy + y\log x + x\cos y = C$
    (b) $xy - y\log x + x\cos y = C$
    (c) $xy + y\log x - x\cos y = C$
    (d) $xy - y\log x - x\sin y = C$

22. If $y = c_1x + c_2.\frac{1}{x}$ is the general solution of the differential equation $x^2\frac{d^2y}{dx^2} + kx\frac{dy}{dx} - y = 0$, for $x > 0$, then k is equal to:
    (a) 1
    (b) –1
    (c) 2
    (d) –2

23. The singular solution of the differential equation $(px - y)^2 = p^2 - 1$ is:
    (a) $x^2 + y^2 + 1 = 0$
    (b) $x^2 + y^2 - 1 = 0$
    (c) $x^2 - y^2 + 1 = 0$
    (d) $x^2 - y^2 - 1 = 0$

24. The Laplace transform of the function $\frac{\sin t}{t}$ i.e., $L\left(\frac{\sin t}{t}\right)$ is -
    (a) $\tan^{-1} s$
    (b) $\tan^{-1}\left(\frac{1}{s}\right)$
    (c) $\frac{1}{s^2+1}$
    (d) $\frac{s}{s^2+1}$

25. The particular integral of the partial differential equation $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$ is:
    (a) $\frac{1}{10}(2\cos(x+2y) + \sin(x+2y))$
    (b) $\frac{1}{10}(\sin(x+2y) - 2\cos(x+2y))$
    (c) $-\frac{1}{10}(\cos(x+2y) + 2\sin(x+2y))$
    (d) $-\frac{1}{10}(\sin(x+2y) - 2\cos(x+2y))$

26. A particle is moving with simple harmonic motion. At the end of three successive seconds, its distance from the mean position measured in the same direction are 1, 5 and 5. The period of complete oscillation is:
    (a) $\frac{\pi}{\cos^{-1}(5/6)}$
    (b) $\frac{\pi}{\cos^{-1}(3/5)}$
    (c) $\frac{2\pi}{\cos^{-1}(5/6)}$
    (d) $\frac{2\pi}{\cos^{-1}(3/5)}$

27. A motor car weighing 500 kg and travelling at 12 m/sec is brought to rest in 18 meters by the application of its brakes. The work done by the force of resistance due to brakes is:
    (a) 18000 J
    (b) 36000 J
    (c) 54000 J
    (d) 72000 J

28. If h and h' are the greatest heights in the two paths of the projectile with a given velocity u for a given range R, then R is given by -
    (a) $4\sqrt{h^2 + h'^2}$
    (b) $\sqrt{2hh'}$
    (c) $4\sqrt{hh'}$
    (d) $16\sqrt{hh'}$

29. A basic feasible solution to the system $AX = b$ of linear equations is said to be degenerate if:
    (a) at most one of the basic variables vanishes
    (b) any one of the basic variables vanishes
    (c) at least one of the basic variables vanishes
    (d) all the basic variables except one vanish

30. If $\overline{u} = \frac{\overline{r}}{r}$, then $\nabla(\nabla.\overline{u})$ is given by -
    (a) $2\overline{r}/r^5$
    (b) $-2\overline{r}/r^5$
    (c) $\frac{2\overline{r}}{r^3}$
    (d) $-\frac{2\overline{r}}{r^3}$

31. Using the Green's theorem in plane, the line integral
    $\int_C [(\cos x \sin y - xy)dx + (\sin x \cos y)dy]$
    over the circle $C: x^2 + y^2 = 1$ is -
    (a) 0
    (b) 2/3
    (c) 4/3
    (d) 8/3

32. If **V** is the volume of the space enclosed by the closed surface **S,** with the unit normal vector $\hat{n}$ pointing outward and $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\iint_S (\nabla r^2).\hat{n}\, dS$ is equal to:
    (a) $\frac{4}{3}V$
    (b) 3V
    (c) 4V
    (d) 6V

33. The set
    $$M = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} : \text{x is a nonzero real number} \right\}$$
    forms a group under matrix multiplication. The inverse of each element $\begin{bmatrix} x & x \\ x & x \end{bmatrix} \in M$ is:
    (a) $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$
    (b) $\begin{bmatrix} 1/x & 1/x \\ 1/x & 1/x \end{bmatrix}$
    (c) $\begin{bmatrix} \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{2x} & \frac{1}{2x} \end{bmatrix}$
    (d) $\begin{bmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{bmatrix}$

34. If a group G has an even number of elements, then apart from the identity element of G:
    (a) at most one element of G is its own inverse.
    (b) At least one element of G is its own inverse.
    (c) precisely one element of G is its own inverse.
    (d) even number of elements of G are their own inverses.

35. If G is a cyclic group of order 8, then the number of generators of G are:
  (a) 1
  (b) 2
  (c) 3
  (d) 4

36. Let **H** and **K** be any two subgroups of a group **G**, each having 12 elements. Which of the following number cannot be the cardinality of the set **HK**?
  (a) 24
  (b) 48
  (c) 60
  (d) 72

37. Let **R** be a commutative ring with unity element 1. Then -
  (a) **R** is an integral domain
  (b) **R** is an integral domain if and only if cancellation laws hold in **R**.
  (c) cancellation laws hold in **R**
  (d) **R** is without zero divisor

38. Let $x_n = 2^{2n}\left(1-\cos\frac{1}{2^n}\right)$ for all $n \in N$.

Then the sequence $<x_n>$:
  (a) does not converge
  (b) converges to 1
  (c) converges to ½
  (d) converges to 0

39. The infinite series $\sum_{n=1}^{\infty}\frac{n+1}{n^p}$ is convergent for:
  (a) $0 < p < 1$
  (b) $1 < p < 2$
  (c) $p = 2$
  (d) $p > 2$

40. The function $f(x)=\begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$

is:
  (a) continuous everywhere
  (b) discontinuous everywhere except at $x=\frac{1}{2}$
  (c) continuous everywhere except at $x=\frac{1}{2}$
  (d) discontinuous everywhere

41. If $f(x) = x^2$ for all $x \in [0,3]$ and $P = \{0,1,2,3\}$ is a partition of $[0,3]$, then $L(P,f)$ and $U(P,f)$ are ______ respectively.
  (a) 5 and 14
  (b) 5 and 13
  (c) 3 and 6
  (d) 3 and 14

42. Every compact subset of the set ***R*** of real numbers is:
  (a) open
  (b) closed
  (c) open and bounded
  (d) closed and bounded

43. The value of the complex integral $\int_{|z|=3}\frac{e^{2z}}{(z+1)^4}dz$ is:
  (a) $\frac{8\pi i}{3}e^{2}$
  (b) $\frac{8\pi i}{3}e^{-2}$
  (c) $\frac{4\pi i}{3}e^{-2}$
  (d) $\frac{16\pi i}{3}e^{-2}$

44. The value of the complex integral $\int_C (\overline{z})^2 dz$, where $C : |z-1| = 1$, is:
   (a) 0
   (b) πi
   (c) $2\pi i$
   (d) $4\pi i$

45. The Laurent's expansion of the complex function $f(z) = \frac{1}{(z+1)(z+3)}$ for the region $0 < |z+1| < 2$ is:
   (a) $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{(z+1)^{n+2}}$
   (b) $\frac{1}{4}\sum_{n=0}^{\infty} \frac{2^n}{(z+1)^{n-2}}$
   (c) $\frac{1}{4}\sum_{n=0}^{\infty} (-1)^n \left(\frac{z+1}{2}\right)^{n-1}$
   (d) $\frac{1}{2}\sum_{n=0}^{\infty} (-1)^n \left(\frac{z+1}{2}\right)^{n-1}$

46. The singularity $z = -1$ of the complex function $f(z) = (z-2)\sin\left(\frac{1}{z+1}\right)$ is:
   (a) a removable singularity
   (b) a pole
   (c) an isolated essential singularity
   (d) a non-isolated essential singularity

47. The approximate value of the integral $\int_0^1 \frac{1}{1+x} dx$ using the Trapezoidal's mid-point rule (n = 2) is -
   (a) 11/24
   (b) 11/36
   (c) 17/24
   (d) 17/36

48. Pick the odd one out:
   (a) Runge-Kutta Method
   (b) Bisection Method
   (c) Method of False Position
   (d) Newton-Raphson Method

49. The first approximate solution to the system
   $8x - 3y + 2z = 20$,
   $6x + 4y + 12z = 35$,
   $4x + 11y - z = 32$
   using the Gauss - Seidel iterative method is:
   (a) $x^{(1)} = 2.5, y^{(1)} = 2.909, z^{(1)} = 2.92$
   (b) $x^{(1)} = 2.5, y^{(1)} = 2, z^{(1)} = 1$
   (c) $x^{(1)} = 2.5, y^{(1)} = 8.75, z^{(1)} = 32$
   (d) $x^{(1)} = 2.5, y^{(1)} = 5, z^{(1)} = 33$

50. The conversion of the binary number $(1.1110011)_2$ to octal system is:
   (a) $(1.163)_8$
   (b) $(4.163)_8$
   (c) $(4.714)_8$
   (d) $(1.714)_8$

PART – II

(Conventional Type Questions)

Answer any 2 (two) from Questions 51 to 55. Each question carries 25 marks.

[2 x 25 = 50]

51.

(a) Compute the eigen values and the associated eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

(b) State and prove the Cauchy's theorem for the complex function $f(z)$.

52.

(a) Find the area bounded by the curve $x = a\cos^3 t,\ y = a\sin^3 t$.

(b) A sphere of constant radius $\boldsymbol{r}$ passes through the origin $\boldsymbol{O}$ and cuts the axes in points $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ respectively. Find the locus of the foot of the perpendicular drawn from $\boldsymbol{O}$ to the plane $\boldsymbol{ABC}$.

53.

(a) Solve the following differential equation by the method of variation of parameters:

$$(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = (1-x)^2$$

(b) Find:

$$L^{-1}\left(\frac{3s+1}{(s-1)(s^2+1)}\right).$$

54.

(a) A stone is projected at an angle $\boldsymbol{\alpha}$ to the horizontal, so as to clear two walls of the equal heights $\boldsymbol{a}$ at a distance $\boldsymbol{2a}$ from each other. Find the range of the projectile.

(b) Prove that $\text{div}\left[\frac{f(r)}{r}\bar{r}\right] = \frac{1}{r^2}\frac{d}{dr}(r^2 f(r))$

55.

(a) Prove that every field is an integral domain.

(b) Prove that every sequence $< x_n >$ is a convergent sequence if and only if it is a Cauchy sequence.

PART - III

(Conventional Type Questions)

Answer any 5 (five) from Questions 56 to 63. Each question carries 10 marks.

[5 x 10 = 50]

56. If $u = (ar^n + br^{-n})(\cos n\theta + \sin n\theta)$, then prove that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$

57. Find the equation of the right circular cylinder whose guiding curve is the circle
$x^2 + y^2 + z^2 = 9;$
$x - y + z = 3.$

58. Solve the following differential equation:
$$(D^2 - 1)y = \cosh x \cos x$$

59. Use Gauss's divergence theorem to evaluate the double integral
$$\iint_S \left[(x+z)\,dydz + (y+z)dzdx + (x+y)dxdy\right],$$
where $\boldsymbol{S}$ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

60. Prove that a subgroup $\boldsymbol{H}$ of a group $\boldsymbol{G}$ is normal if and only if the product of any two right cosets of $\boldsymbol{H}$ in $\boldsymbol{G}$ is again a right coset of $\boldsymbol{H}$ in $\boldsymbol{G}$.

61. Prove that every monotonic function $\boldsymbol{f}$ defined in the interval $[\boldsymbol{a}, \boldsymbol{b}]$ is Riemann-integrable over $[\boldsymbol{a}, \boldsymbol{b}]$.

62. Let $\boldsymbol{a}$ be an isolated singularity of the complex function $\boldsymbol{f}(\boldsymbol{z})$ and $|\boldsymbol{f}(\boldsymbol{z})|$ be bounded in some neighbourhood of $\boldsymbol{a}$. Prove that $\boldsymbol{a}$ is a removable singularity of $\boldsymbol{f}(\boldsymbol{z})$.

63. Find the dual of the following linear programming problem:

$Min.\, z = x_1 + x_2 + x_3$
s.t. $x_1 - 3x_2 + 4x_3 = 5$
$x_1 - 2x_2 \leq 3$
$2x_2 - x_3 \geq 4$
$x_1, x_2 \geq 0; x_3$ *is unrestricted in sign.*

~~~~~~~***~~~~~~~
~~~~~~~